

OLIWIA SZCZUPSKA

Normative Properties of Approval Voting – An Experimental Approach

Abstract

This paper analyzes the principles by which voters extend their preferences over individual candidates to preferences over sets of candidates in the context of approval voting. Under a system of approval voting, a voter is allowed to vote for as many candidates as he wishes and the candidate(s) with the most approval votes wins. We say that a voter has voted sincerely if he prefers all approved above all non-approved candidates. There are certain assumptions regarding the principles of preference extension under which approval voting will never give a voter an incentive to vote insincerely. However, as presented in this paper, the experimental data show that even the most basic principles put forward in the literature are empirically demonstrated for only limited proportion among the examined groups.

Keywords: approval voting, sincere voting, preferences over sets of candidates, elections.

1. Introduction

Approval voting (AV) is a voting procedure that allows each voter to approve of as many candidates as he wishes and the candidate(s) with the most approval votes wins. Its theoretical framework has been introduced by Brams and Fishburn (1978). Being strongly advocated by some and severely criticized by others, AV is an important election system studied in economic theory as well as widely used in practice.

In a standard model of voting a voter has a ranking of the candidates that represents his preferences and votes by reporting this ranking (sincerely or not). In AV a ballot is a subset of candidates. It is called sincere if the voter prefers each of the approved candidates to each of the disapproved ones. As any given preference leads to multiple sincere ballots, there is no single way of voting sincerely. The question being asked is whether there are any reasonable assumptions under which none of the voters would have any incentive to manipulate an AV election by submitting an insincere ballot. In the case of only two candidates a problem of insincere voting does not occur under any reasonable voting procedure. Therefore, we will analyze elections with three or more candidates. The second type of assumption regards a voter's preferences over sets of candidates. With more than one candidate getting a maximal number of approvals AV leads to a set of tied

winners, from which one should be chosen using an appropriate tie-breaking rule. Then, a voter considering manipulating an election has to judge alternative sets of winning candidates and his strategy depends on his preferences over these sets.

Since a voter's preference ranking of the set of candidates is not enough to fully illustrate his preferences over election outcomes (being nonempty sets of candidates), we will consider certain assumptions concerning the principles by which a voter extends his preferences over individual candidates to a preference order over (nonempty) sets of candidates. This is a well-studied problem in social choice theory (see e.g. Barberà et al., 2004; Gärdenfors, 1976; Kannay, Peleg, 1984; Kelly, 1977; Nitzan, Pattanaik, 1984).

The paper is organized as follows. In Section 2 we review a range of properties of how voter's preferences over individual candidates are extended to preferences over sets of candidates as well as present some results on the absence of incentives to vote insincerely under approval voting. In Section 3 an experiment is described evaluating empirical confirmation of these normative properties and the impact the subject of voters' choice has on the results. We summarize experimental results in Section 4 and conclude in Section 5.

2. Normative Properties of Approval Voting

In this section we review a range of axioms proposed in the literature for modeling the problem of extending voter's preferences over individual candidates to preferences over sets of candidates. Additionally, we present theoretical results for elections with a small number of candidates on the absence of incentives to vote insincerely under approval voting for different sets of assumptions on how preferences are extended to set preferences (based on the work of Endriss (2009)).

Each voter has preferences over both individual candidates and nonempty sets of candidates. Let C be a finite set of two or more candidates and $\underline{C} = 2^C \setminus \{\emptyset\}$ denote the set of nonempty sets of candidates. We assume that each voter has a total preference order over individual candidates R defined over C . We will write aRb to express that a certain voter likes candidate a at least as much as candidate b . For the related strict preference relation denoted by P we will write aPb if aRb but not bRa . We also assume that each voter has a weak preference order over nonempty sets of candidates \succeq defined over \underline{C} . We will write $A \succ B$ if $A \succeq B$ but not $B \succeq A$ to account for strict preference and $A \sim B$ if both $A \succeq B$ and $B \succeq A$ to account for indifference.

We will say that a certain axiom is satisfied if the specified conditions hold for a given set of candidates and selection of relations. Variables a, b, \dots range over C and variables A, B, \dots range over \underline{C} and all axioms are being postulated for all choices of a, b, \dots and A, B, \dots from the appropriate ranges.

We will now define a number of axioms that express properties of \succeq , formulated in terms of both R and \succeq :

Kelly Principle¹ (KEL)

It is a conjunction of three fundamental properties. The extension axiom (EXT) states that a voter comparing two singleton sets, should have his set preferences directly corresponding to his preferences over individuals. The maximum axiom (MAX) says that a voter should never prefer a set over a singleton of the best element in that set and the minimum axiom (MIN) expresses that a voter should always weakly prefer a set over a singleton of the worst element in that set (Kelly (1977)). Approaches for extending preferences described below will be enhancements of the Kelly Principle.

$$\left\{ \begin{array}{l} (EXT) \quad \{a\} \succ \{b\} \text{ if } aPb, \\ (MAX) \quad \{\max(A)\} \succeq A, \\ (MIN) \quad A \succeq \{\min(A)\}. \end{array} \right.$$

Where we write $\max(A)$ for the (unique) maximal element in A , i.e. such $a \in A$ that aRb for all $b \in A$ and $\min(A)$ for the (unique) minimal element in A likewise.

Gärdenfors Principle (GAR)

It requires that adding an element which is more (*GF1*)/ less (*GF2*) preferred than all elements in a given set A should lead to a set that is more (*GF1*) / less (*GF2*) preferred than the original one (Gärdenfors, 1976). It is more restrictive than the Kelly Principle.

$$\left\{ \begin{array}{l} (GF1) \quad A \cup \{b\} \succ A \text{ if } bPa \text{ for all } a \in A, \\ (GF2) \quad A \succ A \cup \{b\} \text{ if } aPb \text{ for all } a \in A. \end{array} \right.$$

Kannai-Peleg Principle

It is represented by the independence axiom, which expresses that adding the same extra candidate to two sets should not reverse voter's preferences over these two newly created sets (Kannai, Peleg, 1984).

$$(IND) \quad A \cup \{c\} \succeq B \cup \{c\} \text{ if } A \succ B \text{ and } c \notin A \cup B.$$

Sen-Puppe Principle

It derives from the conjunction of the, introduced earlier, Gärdenfors Principle and the single-flip axiom stating that a set created by replacing a less preferred

¹ We introduce the terminology in the reference to the work of given authors.

candidate with a new, more preferred one should be weakly preferred over the original one² (Puppe, 1995; Sen, 1991).

$$\left\{ \begin{array}{ll} (GF1) & A \cup \{b\} \succ A \quad \text{if } bPa \text{ for all } a \in A, \\ (GF2) & A \succ A \cup \{b\} \quad \text{if } aPb \text{ for all } a \in A, \\ (SIF) & (A \setminus \{a\}) \cup \{b\} \succeq A \quad \text{if } bPa \text{ for all } a \in A \text{ and } b \notin A. \end{array} \right.$$

Nitzan-Pattanaik Principle

It stems from the idea to rank sets using their medians (Nitzan, Pattanaik, 1984) on top of holding Gärdenfors Principle conditions.

$$\left\{ \begin{array}{ll} (GF1) & A \cup \{b\} \succ A \quad \text{if } bPa \text{ for all } a \in A, \\ (GF2) & A \succ A \cup \{b\} \quad \text{if } aPb \text{ for all } a \in A, \\ (MED) & A \sim med(A). \end{array} \right.$$

Where formally median of a set is defined as following subset:

$$med(A) = \{a \in A : \#\{b \in A : bRa\} - \#\{b \in A : aRb\} \in \{0, 1, -1\}\}.$$

We will now present some assumptions of how voters extend their preferences to preferences over sets of candidates under which AV will never incentivize voters to vote insincerely. The following theoretical results have been established by Endriss in ‘Sincerity and Manipulation under Approval Voting’ paper (Endriss, 2009). Due to the nature of conducted experiment we will focus on the case of three-candidate elections.

Following Endriss’ approach, we assume our possible manipulator knows votes of all others. Given this, points obtained by each candidate can be calculated. The candidate (one or more in case the highest number is shared) who receives the most approvals will be called pivotal. Those with precisely one point less will be called subpivotal and the remaining ones insignificant. None of insignificant candidates will have a chance of being elected regardless of manipulator’s vote as via his ballot he can only lead to the election of a subset of some pivotal or subset of all the pivotal and some subpivotal candidates. We will consider all possible combinations of types of candidates, i.e. at least one of them being pivotal. In each situation our manipulator chooses to vote the way he achieves the best outcome(s) possible. Depending on the assumptions made on the preference extension, we can reduce the number of situations in which insincere manipulation can arise, leading to the point where any strategy (in particular insincere) is weakly dominated by some sincere strategy.

² The (SIF) axiom formulated here is a simpler version pointed put by Puppe as an equivalent to the one originally suggested by Sen, i.e. that A should be weakly preferred to B , if the two sets have the same cardinality and there exists a surjective mapping $f : A \rightarrow B$ such that $aRf(a)$ for all $a \in A$.

Below we present Endriss' positive results of this sort for a three-candidate election in particular.

Theorem 1. *Under AV with three candidates, any strategy available to a voter who knows all other ballots and whose preferences satisfy Kelly Principle (KEL), (AX1), and (AX2)³ is weakly dominated by some sincere strategy*

$$\left\{ \begin{array}{ll} \text{(AX1)} & \{a, b\} \succeq \{a, b, c\} & \text{if } aPb \text{ and } bPc, \\ \text{(AX2)} & \{a, c\} \succeq \{b, c\} \text{ or } \{a, b, c\} \succeq \{b, c\} & \text{if } aPb \text{ and } bPc. \end{array} \right.$$

Theorem 2. *Under AV with three candidates, any strategy available to a voter who knows all other ballots and whose preferences satisfy Gärdenfors Principle (GAR) is weakly dominated by some sincere strategy.*

3. Experimental Design

In this section we describe an experiment designed to empirically verify presented theoretical results. It was conducted on three independent groups of Warsaw School of Economics students in Q1 2012. The participants were presented with a questionnaire. There were three candidates listed in the first part and seven sets of these candidates listed in the second part. Each of the students was asked to rank three alternatives in order of preference (assigning the same rank for different candidates was not allowed) in the first part, and rank all possible sets of these alternatives in order of preference (assigning the same rank for different sets was allowed) in the second part. They did not contact each other. Total of $N = 281$ questionnaires were conducted. Depending on the group, with the aim to cover political, social and consumer choices, the subject were:

- (A) websites, $N_A = 96$, $C_A = \{\text{facebook.com, onet.pl, pudelek.pl}\}$;
- (B) politicians, $N_B = 92$, $C_B = \{\text{Kaczyński, J.Palikot, D.Tusk}\}$;
- (C) brands of beer, $N_C = 93$, $C_C = \{\text{Lech, Tyskie, Żywiec}\}$.

Particular alternatives were earlier decided among a peer group of students as representing highly popular options among Polish society, ensuring awareness and making it interesting for the participants.

The described experimental method enabled us to gather information on both ranks of individual candidates and sets of candidates that we then confronted with theoretical assumptions on preference extension described in the previous section.

³ These two plain axioms represent the minimal additional assumptions we need to make on top of the Kelly Principle, if we want to rule out any incentives for our would-be manipulator to vote insincerely. For an in-depth systematical analysis of the case of three candidates please refer to the work of Endriss (2009).

Figure 1. Questionnaire used in the experiment – brands of beer example

(1) Please rank below brands of beer on the 1-3 scale in order of your preference, where 1 – most preferred brand. You cannot assign the same rank for two different brands.							
<i>Lech</i>	1	2	3				
<i>Tyskie</i>	1	2	3				
<i>Żywiec</i>	1	2	3				
(2) Now, I would ask you to rank below sets of brands on the 1-7 scale in order of your preference, where 1 – most preferred set. You can assign the same rank to more than one set.							
{ <i>Lech</i> }	1	2	3	4	5	6	7
{ <i>Tyskie</i> }	1	2	3	4	5	6	7
{ <i>Żywiec</i> }	1	2	3	4	5	6	7
{ <i>Lech, Tyskie</i> }	1	2	3	4	5	6	7
{ <i>Lech, Żywiec</i> }	1	2	3	4	5	6	7
{ <i>Tyskie, Żywiec</i> }	1	2	3	4	5	6	7
{ <i>Lech, Tyskie, Żywiec</i> }	1	2	3	4	5	6	7

To answer the question about the extent to which certain normative properties of approval voting are empirically demonstrated we analyzed the data on an aggregated level ($N = 281$) by verifying for each individual participant’s ballot whether certain axioms and principles are met. To answer the question about the impact the subject of voters’ choice (i.e. websites, politicians or brands of beers respectively) has on the results we ran similar exercise on each of the three groups separately.

4. Results

Results presented in this section demonstrate that the principles by which voters are supposed to extend their preferences over individual candidates to preferences over sets of candidates are experimentally confirmed only for limited proportion of tested group and, furthermore, that there are significant differences observed depending on the subject of choice.

Table 1. Axioms satisfied among tested groups combined (%); N=281

EXT	MAX	MIN	GF1	GF2	IND	SIF	MED	AX1	AX2
71.9	60.9	50.2	24.6	31.7	38.1	70.5	11.0	74.4	82.6

As presented in table 1, only four of the key axioms considered are being satisfied for more than half of all the subjects tested. And taking into account how fundamental they seem (as described in Section 2), we might have still expected the incidences to be higher. The remaining, more restrictive axioms are satisfied for less than 40% of voters.

Table 2. Principles satisfied among tested groups combined (%); N=281

<i>Kelly</i>	Theorem 1	<i>Gärdenfors</i> Theorem 2	<i>Kannai- Peleg</i>	<i>Sen- Puppe</i>	<i>Nitzan- Pattanaik</i>
EXT&MAX&MIN	KEL&AX1&AX2	GF1&GF2	IND	GAR&SIF	GAR&MED
20.3	18.9	5.3	38.1	5.3	0.4

When analyzing the principles and theorems, where certain axioms occur in conjunction with others, the incidence of subjects among whom the properties are being satisfied decreases even further. Theorem 1 states that in a three-candidate case, if the given two plain axioms hold on top of the Kelly Principle, the voter would not have the incentive to vote insincerely under approval voting. However (table 2), it has been experimentally confirmed for less than one-fifth of the tested group.

Interesting results could be observed by analyzing the data by different subjects of voters' choice: social (websites), political (politicians) and consumer (beers).

The null hypotheses of equality of proportions (for which certain axioms or principles are satisfied) have been verified in a pairwise manner among tested groups. Below we present the results of two-tailed two-proportion z -tests at $\alpha = 0.05$ significance level. In cases where we reject the null hypothesis, a superscript letter A/B/C denotes a relatively higher proportion compared to the group for which A/B/C stands for.

For the group choosing among politicians (table 3), six out of eight key axioms are being satisfied for significantly lower proportions compared to either of the two other groups (it is only in the case of the extension and median axiom where these incidences come closer). There are also several differences observed between the social and consumer subject of choice, with the latter noting a significantly higher incidence for only one of the axioms (maximum).

In the case of more complex principles the impact of the subject of choice is even more visible. When it comes to satisfying Kelly Principle and Theorem 1, politicians' group notes statistically significantly lower incidence vs. both other groups. The theoretical concept of ruling out insincere voting in the case of choosing politicians is empirically demonstrated for only a small proportion of the tested group (6.5%), and even with the incidence growing substantially for consumer goods choice (17%) and further up for websites, it doesn't reach beyond one-third for any of the tested groups.

One can assume that choosing a politician should be a more important decision for a voter, hence made more carefully and with greater consistency than when it comes to websites or beer. However, the results show differently.

Table 3. Axioms satisfied across tested groups (%); $N_A = 96$, $N_B = 92$, $N_C = 93$. Superscript letter denotes significantly higher proportion vs. respective group A/B/C

	EXT	MAX	MIN	GF1	GF2	IND	SIF	MED	AX1	AX2
A Websites	81.3 ^B	59.4	79.2 ^{BC}	52.1 ^{BC}	33.3 ^B	58.3 ^B	92.7 ^{BC}	11.5	76.0	97.9 ^{BC}
B Politicians	70.7	48.9	23.9	5.4	18.5	8.7	44.6	12.0	63.0	60.9
C Beers	63.4	74.2 ^{AB}	46.2 ^B	15.1 ^B	43.0 ^B	46.2 ^B	73.1 ^B	9.7	83.9 ^B	88.2 ^B

Table 4. Principles satisfied across tested groups (%); $N_A = 96$, $N_B = 92$, $N_C = 93$. Superscript letter denotes significantly higher proportion vs. respective group A/B/C

	<i>Kelly</i>	Theorem 1	<i>Gärdenfors</i> Theorem 2	<i>Kannai- Peleg</i>	<i>Sen- Puppe</i>	<i>Nitzan- Pattanaik</i>
A Websites	EXT&MAX&MIN 32.3 ^B	KEL&AX1&AX2 32.3 ^{BC}	GF1&GF2 10.4 ^B	IND 58.3 ^B	GAR&SIF 10.4 ^B	GAR&MED 1.0
B Politicians	6.5	6.5	2.2	8.7	2.2	0.0
C Beers	21.5 ^B	17.2 ^B	3.2	46.2 ^B	3.2	0.0

Possible explanation of this pattern could be that politicians seem to be the most polarizing subject of voting compared to the others tested, i.e. the percentage of voters ranking the set of all three alternatives at least as high as the set of two or a single candidate is 2% for politicians vs. 31% and 10% for websites and beer respectively.

5. Conclusions

The aim of this paper was to verify whether some of the normative properties regarding the extension preferences over individual candidates to preferences over sets of candidates may be confirmed experimentally. The answer for a three-candidate case could be perceived as positive only for the most basic fundamental axioms, whereas definitely negative for more complex principles, in particular theorems under which approval voting is proven to never give a voter an incentive to vote insincerely. Thus, the promising theoretical results on AV strategy-proofness fail at successfully translating into practical implications.

Furthermore, we conclude that these results are significantly impacted by the subject of voters' choice. In particular, for political choice most of the properties were satisfied for relatively low proportions compared to social and consumer choice.

The results have preliminary character and need deeper analysis. With regards to assumptions ruling out insincere voting, it is worth noting that when increasing the number of candidates the number of additional plain axioms that we would have to accept would go up as well. Thus, we would presumably observe similar negative results to even larger extent.

Bibliography

- [1] Barberà S., Bossert W., Pattanaik P.K. (2004), *Ranking sets of objects*, *Handbook of Utility Theory*, Vol. 2, Kluwer Academic Publishers.
- [2] Brams S.J., Fishburn P.C. (1978), *Approval voting*, "The American Political Science Review", 72 (3), pp. 831-847.
- [3] Endriss U. (2009), *Sincerity and Manipulation under Approval Voting*, Working Paper 29 October 2009, Institute for Logic, Language and Computation University of Amsterdam,
<http://staff.science.uva.nl/~ulle/pubs/working-papers/sincere-approval.pdf>.
- [4] Gärdenfors P. (1975), *Manipulation of social choice functions*, "Journal of Economic Theory", 13 (2), pp. 217-228.
- [5] Kannai Y., Peleg B. (1984), *A note on the extension of an order on a set to the power set*, "Journal of Economic Theory", 32 (1), pp. 172-175.

- [6] Kelly J. (1977), *Strategy-proofness and social choice functions without single-valuedness*, "Econometrica", 45 (2), pp. 439-446.
- [7] Nitzan S.I., Pattanaik P.K. (1984), *Median-based extensions of an ordering over a set to the power set: An axiomatic characterization*, "Journal of Economic Theory", 34 (2), pp. 252-261.
- [8] Puppe C. (1995), *Freedom of choice and rational decisions*, "Social Choice and Welfare", 12 (2), pp. 137-153.
- [9] Sen A.K. (1991), *Welfare, preference and freedom*, "Journal of Econometrics", 50 (1-2), pp. 15-29.

Własności normatywne głosowania aprobowującego – podejście eksperymentalne

Abstrakt

W pracy przeanalizowano zasady rozszerzania preferencji na pojedynczych kandydatach do preferencji na zbiorach kandydatów w kontekście głosowania aprobowującego. W głosowaniu aprobowującym głosujący oddaje głos na dowolną liczbę kandydatów i ten/ci z największą liczbą głosów zwycięża. O głosującym powiemy, że głosuje szczerze jeśli preferuje wszystkich zaaprobowanych przez siebie kandydatów nad wszystkich niezaaprobowanych. W teorii istnieją pewne założenia dotyczące wspomnianych zasad rozszerzania preferencji, których spełnienie gwarantuje brak motywacji do głosowania strategicznego. Natomiast, jak pokazano w artykule, dane empiryczne wskazują, że nawet najbardziej podstawowe z tych proponowanych w literaturze zasad są empirycznie potwierdzone jedynie dla niewielkiego odsetka spośród badanej grupy.

Słowa kluczowe: głosowanie aprobowujące, głosowanie strategiczne, preferencje na zbiorach kandydatów, wybory.

Author:

Oliwia Szczupska, Collegium of Economic Analysis, Warsaw School of Economics,
Al. Niepodległości 162, 02-554 Warsaw, Poland,
email: oliwia.szczupska@gmail.com